Solving Inequalities Using Multiplication or Division

Essential Question How can you use division to solve an inequality?

EXPLORATION 1 **Writing a Rule**

Work with a partner.

LOOKING FOR A PATTERN

To be proficient in math, you need to investigate relationships, observe patterns, and use your observations to write general rules.

a. Copy and complete the table. Decide which graph represents the solution of the inequality $6 < 3x$. Write the solution of the inequality.

x	-1	0	1	2	3	4	5
$3x$	-3						
$6 \overset{?}{<} 3x$	No						

b. Use a table to solve each inequality. Then write a rule that describes how to use division to solve the inequalities.

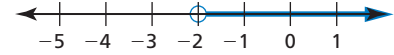**i.** $2x < 4$ **ii.** $3 \geq 3x$ **iii.** $2x < 8$ **iv.** $6 \geq 3x$

EXPLORATION 2 **Writing a Rule**

Work with a partner.

a. Copy and complete the table. Decide which graph represents the solution of the inequality $6 < -3x$. Write the solution of the inequality.

x	-5	-4	-3	-2	-1	0	1
$-3x$							
$6 \overset{?}{<} -3x$							

b. Use a table to solve each inequality. Then write a rule that describes how to use division to solve the inequalities.

i. $-2x < 4$ **ii.** $3 \geq -3x$ **iii.** $-2x < 8$ **iv.** $6 \geq -3x$

Communicate Your Answer

3. How can you use division to solve an inequality?

4. Use the rules you wrote in Explorations 1(b) and 2(b) to solve each inequality.

     a. $7x < -21$ **b.** $12 \leq 4x$ **c.** $10 < -5x$ **d.** $-3x \leq 0$

What You Will Learn

▶ Solve inequalities by multiplying or dividing by *positive* numbers.

▶ Solve inequalities by multiplying or dividing by *negative* numbers.

▶ Use inequalities to solve real-life problems.

Multiplying or Dividing by Positive Numbers

🔵 Core Concept

Multiplication and Division Properties of Inequality (c > 0)

Words Multiplying or dividing each side of an inequality by the same *positive* number produces an equivalent inequality.

Numbers

$$-6 < 8 \qquad\qquad 6 > -8$$

$$2 \cdot (-6) < 2 \cdot 8 \qquad \frac{6}{2} > \frac{-8}{2}$$

$$-12 < 16 \qquad\qquad 3 > -4$$

Algebra If $a > b$ and $c > 0$, then $ac > bc$. If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.

If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1 **Multiplying or Dividing by Positive Numbers**

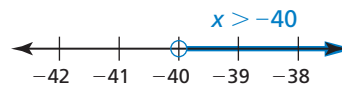Solve (a) $\frac{x}{8} > -5$ and (b) $-24 \geq 3x$. Graph each solution.

SOLUTION

a. $\qquad \frac{x}{8} > -5$ — Write the inequality.

Multiplication Property of Inequality → $8 \cdot \frac{x}{8} > 8 \cdot (-5)$ — Multiply each side by 8.

$\qquad x > -40$ — Simplify.

▶ The solution is $x > -40$.

$x > -40$
$$\xleftarrow{\qquad} \overset{|}{-42} \;\; \overset{|}{-41} \;\; \overset{\oplus}{-40} \;\; \overset{|}{-39} \;\; \overset{|}{-38} \xrightarrow{\qquad}$$

b. $\qquad -24 \geq 3x$ — Write the inequality.

Division Property of Inequality → $\frac{-24}{3} \geq \frac{3x}{3}$ — Divide each side by 3.

$\qquad -8 \geq x$ — Simplify.

▶ The solution is $x \leq -8$.

$x \leq -8$
$$\xleftarrow{\qquad} \overset{|}{-10} \;\; \overset{|}{-9} \;\; \overset{\bullet}{-8} \;\; \overset{|}{-7} \;\; \overset{|}{-6} \xrightarrow{\qquad}$$

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Solve the inequality. Graph the solution.

1. $\frac{n}{7} \geq -1$ **2.** $-6.4 \geq \frac{1}{5}w$ **3.** $4b \geq 36$ **4.** $-18 > 1.5q$

Multiplying or Dividing by Negative Numbers

🅖 Core Concept

Multiplication and Division Properties of Inequality ($c < 0$)

Words When multiplying or dividing each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed to produce an equivalent inequality.

Numbers

$$-6 < 8 \qquad\qquad 6 > -8$$

$$-2 \cdot (-6) > -2 \cdot 8 \qquad \frac{6}{-2} < \frac{-8}{-2}$$

$$12 > -16 \qquad\qquad -3 < 4$$

Algebra If $a > b$ and $c < 0$, then $ac < bc$. If $a > b$ and $c < 0$, then $\dfrac{a}{c} < \dfrac{b}{c}$.

If $a < b$ and $c < 0$, then $ac > bc$. If $a < b$ and $c < 0$, then $\dfrac{a}{c} > \dfrac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

COMMON ERROR

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol, as shown in Example 1.

Only reverse the inequality symbol when you multiply or divide each side by a negative number.

EXAMPLE 2 **Multiplying or Dividing by Negative Numbers**

Solve each inequality. Graph each solution.

a. $2 < \dfrac{y}{-3}$

b. $-7y \leq -35$

SOLUTION

a. $\qquad 2 < \dfrac{y}{-3}$ $\qquad\qquad$ Write the inequality.

Multiplication Property of Inequality → $-3 \cdot 2 > -3 \cdot \dfrac{y}{-3}$ $\qquad$ Multiply each side by -3. Reverse the inequality symbol.

$\qquad\qquad -6 > y$ $\qquad\qquad$ Simplify.

▶ The solution is $y < -6$.

$y < -6$

$\begin{array}{c} \leftarrow\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\! \text{—}\!\!\text{—}\!\!\text{—}\!\!\oplus\!\!\text{—}\!\!\text{—}\!\!\rightarrow \\ -8 \quad -7 \quad -6 \quad -5 \quad -4 \end{array}$
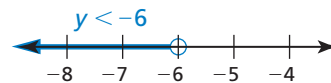
b. $-7y \leq -35$ $\qquad\qquad$ Write the inequality.

Division Property of Inequality → $\dfrac{-7y}{-7} \geq \dfrac{-35}{-7}$ $\qquad$ Divide each side by -7. Reverse the inequality symbol.

$\qquad\qquad y \geq 5$ $\qquad\qquad$ Simplify.

▶ The solution is $y \geq 5$.
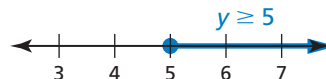
$y \geq 5$

$\begin{array}{c} \text{—}\!\!\text{—}\!\!\text{—}\!\!\bullet\!\!\text{—}\!\!\text{—}\!\!\rightarrow \\ 3 \quad 4 \quad 5 \quad 6 \quad 7 \end{array}$

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Solve the inequality. Graph the solution.

5. $\dfrac{p}{-4} < 7$

6. $\dfrac{x}{-5} \leq -5$

7. $1 \geq -\dfrac{1}{10}z$

8. $-9m > 63$

9. $-2r \geq -22$

10. $-0.4y \geq -12$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

You earn $9.50 per hour at your summer job. Write and solve an inequality that represents the numbers of hours you need to work to buy a digital camera that costs $247.

SOLUTION

1. **Understand the Problem** You know your hourly wage and the cost of the digital camera. You are asked to write and solve an inequality that represents the numbers of hours you need to work to buy the digital camera.

2. **Make a Plan** Use a verbal model to write an inequality. Then solve the inequality.

3. **Solve the Problem**

   Words   Hourly wage · Hours worked ≥ Cost of camera

   Variable Let n be the number of hours worked.

   Inequality 9.5 · n ≥ 247

   $$9.5n \geq 247 \qquad \text{Write the inequality.}$$

   Division Property of Inequality →

   $$\frac{9.5n}{9.5} \geq \frac{247}{9.5} \qquad \text{Divide each side by 9.5.}$$

   $$n \geq 26 \qquad \text{Simplify.}$$

   ▶ You need to work at least 26 hours for your gross pay to be at least $247. If you have payroll deductions, such as Social Security taxes, you need to work more than 26 hours.

REMEMBER

Compatible numbers are numbers that are easy to compute mentally.

4. **Look Back** You can use estimation to check that your answer is reasonable.

   $247 ÷ $9.50/h

   ↓   ↓

   $250 ÷ $10/h = 25 h   Use compatible numbers.

   Your hourly wage is about $10 per hour. So, to earn about $250, you need to work about 25 hours.

Unit Analysis Each time you set up an equation or inequality to represent a real-life problem, be sure to check that the units balance.

$$\frac{\$9.50}{\cancel{h}} \times 26\,\cancel{h} = \$247$$

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11. You have at most $3.65 to make copies. Each copy costs $0.25. Write and solve an inequality that represents the numbers of copies you can make.

12. The maximum speed limit for a school bus is 55 miles per hour. Write and solve an inequality that represents the numbers of hours it takes to travel 165 miles in a school bus.

Vocabulary and Core Concept Check

1. **WRITING** Explain how solving $2x < -8$ is different from solving $-2x < 8$.

2. **OPEN-ENDED** Write an inequality that is solved using the Division Property of Inequality where the inequality symbol needs to be reversed.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the inequality. Graph the solution. *(See Example 1.)*

3. $4x < 8$

4. $3y \le -9$

5. $-20 \le 10n$

6. $35 < 7t$

7. $\dfrac{x}{2} > -2$

8. $\dfrac{a}{4} < 10.2$

9. $20 \ge \dfrac{4}{5}w$

10. $-16 \le \dfrac{8}{3}t$

In Exercises 11–18, solve the inequality. Graph the solution. *(See Example 2.)*

11. $-6t < 12$

12. $-9y > 9$

13. $-10 \ge -2z$

14. $-15 \le -3c$

15. $\dfrac{n}{-3} \ge 1$

16. $\dfrac{w}{-5} \le 16$

17. $-8 < -\dfrac{1}{4}m$

18. $-6 > -\dfrac{2}{3}y$

19. **MODELING WITH MATHEMATICS** You have $12 to buy five goldfish for your new fish tank. Write and solve an inequality that represents the prices you can pay per fish. *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** A weather forecaster predicts that the temperature in Antarctica will decrease 8°F each hour for the next 6 hours. Write and solve an inequality to determine how many hours it will take for the temperature to drop at least 36°F.

USING TOOLS In Exercises 21–26, solve the inequality. Use a graphing calculator to verify your answer.

21. $36 < 3y$

22. $17v \ge 51$

23. $2 \le -\dfrac{2}{9}x$

24. $4 > \dfrac{n}{-4}$

25. $2x > \dfrac{3}{4}$

26. $1.1y < 4.4$

ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in solving the inequality.

27.

$$-6 > \dfrac{2}{3}x$$

$$\dfrac{3}{2} \cdot (-6) < \dfrac{3}{2} \cdot \dfrac{2}{3}x$$

$$-\dfrac{18}{2} < x$$

$$-9 < x$$

The solution is $x > -9$.

28.

$$-4y \le -32$$

$$\dfrac{-4y}{-4} \le \dfrac{-32}{-4}$$

$$y \le 8$$

The solution is $y \le 8$.

29. **ATTENDING TO PRECISION** You have $700 to buy new carpet for your bedroom. Write and solve an inequality that represents the costs per square foot that you can pay for the new carpet. Specify the units of measure in each step.
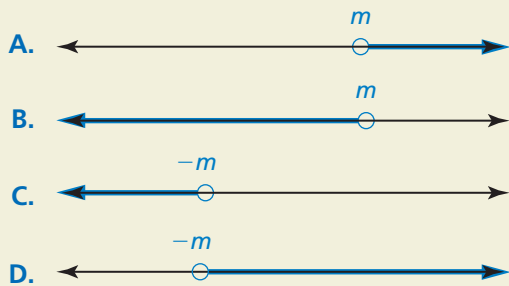
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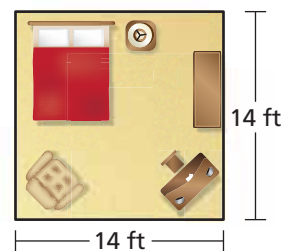
30. HOW DO YOU SEE IT? Let $m > 0$. Match each inequality with its graph. Explain your reasoning.

a. $\dfrac{x}{m} < -1$ **b.** $\dfrac{x}{m} > 1$

c. $\dfrac{x}{m} < 1$ **d.** $-\dfrac{x}{m} < 1$

A.

B.

C.

D.

31. MAKING AN ARGUMENT You run for 2 hours at a speed no faster than 6.3 miles per hour.

a. Write and solve an inequality that represents the possible numbers of miles you run.

b. A marathon is approximately 26.2 miles. Your friend says that if you continue to run at this speed, you will not be able to complete a marathon in less than 4 hours. Is your friend correct? Explain.

32. THOUGHT PROVOKING The inequality $\dfrac{x}{4} \le 5$ has a solution of $x = p$. Write a second inequality that also has a solution of $x = p$.

33. PROBLEM SOLVING The U.S. Mint pays $0.02 to produce every penny. How many pennies are produced when the U.S. Mint pays more than $6 million in production costs?

**34. REASONING** Are $x \le \dfrac{2}{3}$ and $-3x \le -2$ equivalent? Explain your reasoning.

35. ANALYZING RELATIONSHIPS Consider the number line shown.
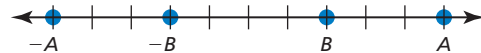
a. Write an inequality relating A and B.

b. Write an inequality relating $-A$ and $-B$.

c. Use the results from parts (a) and (b) to explain why the direction of the inequality symbol must be reversed when multiplying or dividing each side of an inequality by the same negative number.

36. REASONING Why might solving the inequality $\dfrac{4}{x} \ge 2$ by multiplying each side by x lead to an error? (*Hint:* Consider $x > 0$ and $x < 0$.)

37. MATHEMATICAL CONNECTIONS The radius of a circle is represented by the formula $r = \dfrac{C}{2\pi}$. Write and solve an inequality that represents the possible circumferences C of the circle.
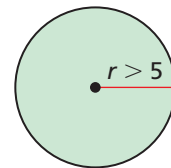
38. CRITICAL THINKING A water-skiing instructor recommends that a boat pulling a beginning skier has a speed less than 18 miles per hour. Write and solve an inequality that represents the possible distances d (in miles) that a beginner can travel in 45 minutes of practice time.

39. CRITICAL THINKING A local zoo employs 36 people to take care of the animals each day. At most, 24 of the employees work full time. Write and solve an inequality that represents the fraction of employees who work part time. Graph the solution.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.2 and Section 1.3)*

40. $5x + 3 = 13$ **41.** $\dfrac{1}{2}y - 8 = -10$

42. $-3n + 2 = 2n - 3$ **43.** $\dfrac{1}{2}z + 4 = \dfrac{5}{2}z - 8$

Tell which number is greater. *(Skills Review Handbook)*

44. $0.8, 85\%$ **45.** $\dfrac{16}{30}, 50\%$ **46.** $120\%, 0.12$ **47.** $60\%, \dfrac{2}{3}$